

Hong Kong Mathematics Olympiad (2014 / 2015)

Final Event 1 (Individual)

香港数学竞赛 (2014 / 2015)

决赛项目 1 (个人)

除非特别声明，答案须用数字表达，并化至最简。

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 若  $|x+\sqrt{5}|+|y-\sqrt{5}|+|z|=0$ ，求  $\alpha = x^2 + y^2 + z^2$ 。

If  $|x+\sqrt{5}|+|y-\sqrt{5}|+|z|=0$ , determine  $\alpha = x^2 + y^2 + z^2$ .

$\alpha =$

2. 若  $\beta$  为乘积  $\underbrace{11111\cdots 11}_{\alpha \text{ 个 } 1} \times \underbrace{99999\cdots 99}_{\alpha \text{ 个 } 9}$  所有数字的数字之和，求  $\beta$  的值。

If  $\beta$  is the sum of all digits of the product  $\underbrace{11111\cdots 11}_{\alpha \text{ 1s}} \times \underbrace{99999\cdots 99}_{\alpha \text{ 9s}}$ , determine the value of  $\beta$ .

$\beta =$

3. 设实函数  $f(x)$  对于所有实数  $x$  及  $y$  满足  $f(xy) = f(x)f(y)$ ，且  $f(1) < 1$ 。

求  $\gamma = f(\beta) + 100 - \beta$  的值。

Suppose that the real function  $f(x)$  satisfies  $f(xy) = f(x)f(y)$  for all real numbers  $x$  and  $y$ , and  $f(1) < 1$ . Determine the value of  $\gamma = f(\beta) + 100 - \beta$ .

$\gamma =$

4. 若  $n$  为正整数及  $f(n) = 2^n + 2^{n-1} + 2^{n-2} + \cdots + 2^2 + 2^1 + 1$ ，求  $\delta = f(\gamma)$  的值。

If  $n$  is a positive integer and  $f(n) = 2^n + 2^{n-1} + 2^{n-2} + \cdots + 2^2 + 2^1 + 1$ , determine the value of  $\delta = f(\gamma)$ .

$\delta =$

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Final Event 2 (Individual)

香港数学竞赛 (2014 / 2015)

决赛项目 2 (个人)

除非特别声明，答案须用数字表达，并化至最简。

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 若  $x_0, y_0, z_0$  为以下方程组的解，求  $\alpha = x_0 + y_0 + z_0$  的值。

If  $x_0, y_0, z_0$  is a solution to the simultaneous equations below, determine the value of  $\alpha = x_0 + y_0 + z_0$ .

$$\begin{cases} x - y - z = -1 \\ y - x - z = -2 \\ z - x - y = -4 \end{cases}$$

$\alpha =$

2. 若  $\beta$  为  $\underbrace{111 \cdots 111}_{100 \text{ 个 } 1} \div \alpha$  的余数，求  $\beta$  的值。

If  $\beta$  is the remainder of  $\underbrace{111 \cdots 111}_{100 \text{ 1s}} \div \alpha$ , determine the value of  $\beta$ .

$\beta =$

3. 若  $\gamma$  为  $\left[ (\beta - 2)^{100} + \beta^{50} + (\beta + 2)^{25} \right] \div 3$  的余数，求  $\gamma$  的值。

If  $\gamma$  is the remainder of  $\left[ (\beta - 2)^{100} + \beta^{50} + (\beta + 2)^{25} \right] \div 3$  determine the value of  $\gamma$ .

$\gamma =$

4. 若方程  $x^4 + ax^2 + bx + \delta = 0$  有四实根，且已知其中三个为 1、 $\gamma$  及  $\gamma^2$ ，求  $\delta$  的值。

If the equation  $x^4 + ax^2 + bx + \delta = 0$  has four real roots with three of them being 1,  $\gamma$  and  $\gamma^2$ , determine the value of  $\delta$ .

$\delta =$

Hong Kong Mathematics Olympiad (2014 / 2015)

Final Event 3 (Individual)

香港数学竞赛 (2014 / 2015)

决赛项目 3 (个人)

除非特别声明，答案须用数字表达，并化至最简。

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 由 1 至 1000 的正整数中，有多少个不能被 5 或 7 整除？

Of the positive integers from 1 to 1000, how many are not divisible by 5 or not divisible by 7?

$\alpha =$

2. 求  $\beta = 1^2 - 2^2 + 3^2 - 4^2 + \dots + \alpha^2$  的值。

Determine the value of  $\beta = 1^2 - 2^2 + 3^2 - 4^2 + \dots + \alpha^2$ .

$\beta =$

3. 求当  $\beta$  除以以下数列中的第 1993 项时的余数。

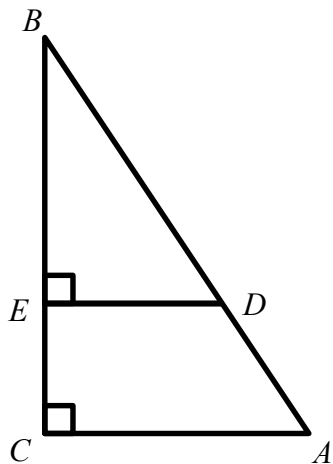
Determine the remainder of  $\beta$  divided by the 1993<sup>rd</sup> term of the following sequence.

1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, ....

$\gamma =$

4. 在下图中,  $BE = AC$ 、 $BD = \frac{1}{2}$  及  $DE + BC = 1$ 。若  $\delta$  是  $ED$  的长度的  $\gamma$  倍, 求  $\delta$  的值。

In the figure below,  $BE = AC$ ,  $BD = \frac{1}{2}$  and  $DE + BC = 1$ . If  $\delta$  is  $\gamma$  times the length of  $ED$ , determine the value of  $\delta$ .



$\delta =$

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Final Event 4 (Individual)

香港数学竞赛 (2014 / 2015)

决赛项目 4 (个人)

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Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 设  $\alpha$  为  $2^{1000}$  除以 13 的余数，求  $\alpha$  的值。

Let  $\alpha$  be the remainder of  $2^{1000}$  divided by 13, determine the value of  $\alpha$ .

$\alpha =$

2. 求  $\beta = \frac{(7+4\sqrt{\alpha})^{\frac{1}{2}} - (7-4\sqrt{\alpha})^{\frac{1}{2}}}{\sqrt{\alpha}}$  的值。

Determine the value of  $\beta = \frac{(7+4\sqrt{\alpha})^{\frac{1}{2}} - (7-4\sqrt{\alpha})^{\frac{1}{2}}}{\sqrt{\alpha}}$ .

$\beta =$

3. 若  $f(a) = a - \beta$  且  $F(a, b) = b^2 + a$ ，求  $\gamma = F(3, f(4))$  的值。

If  $f(a) = a - \beta$  and  $F(a, b) = b^2 + a$ , determine the value of  $\gamma = F(3, f(4))$ .

$\gamma =$

4. 若  $\delta$  是方程  $x^{\log_{\gamma} x} = 10$  所有实根的积，求  $\delta$  的值。

If  $\delta$  is the product of all real roots of  $x^{\log_{\gamma} x} = 10$ , determine the value of  $\delta$ .

$\delta =$